Similarly to the expressions (11) and (15), one can find from (18) the coordinate of the concentration maximum of the second radionucleid of the set as well as the appearance time of the concentration maximum at the adsorber outlet.

One should mention, in conclusion, that the results obtained in this article can be employed in the analysis of radionucleid migration in the earth and also to investigate radioactive gases by chromatography.

NOTATION

$c_i(x, t)$, concentration of the i-th radionucleid in flow; $a_i(x, t)$, concentration of the i-th nucleid in adsorbent; γ, reciprocal of Henri coefficient; λ_i, decay constant of the i-th radionucleid; D, diffusion coefficient; ν, velocity of the main gas carrier.

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ACOUSTIC METHOD OF INVESTIGATING NONSTATIONARY HEAT CONVECTION IN CYLINDRICAL LAYERS OF GASES AND LIQUIDS

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A new acoustic method of investigating nonstationary heat transfer and heat conduction in gases and liquids is described. The apparatus is described and experimental results for air are given.

The hot-wire method or, as it is sometimes called, the cylindrical probe of constant power method [1-5, 14], is widely used to study heat transfer in liquids and gases. Its advantage is the relative simplicity of the measuring cell. This method has also been used to investigate heat conduction and heat convection under steady conditions. The main features of nonstationary free convection have not been studied to any great extent [6-9, 15, 16].

In this paper we use the hot-wire nonstationary acoustic method to investigate the heat-transfer properties of liquids and gases. The method is based on measuring the phase difference or frequency difference (for high heating speeds) of ultrasonic oscillations in fine wires [10-11]. The same wire serves both as a heater and for measuring the temperature.

The response time of the measuring probe, determined by the time taken for the acoustic signal to propagate through the control part of the medium, is of the order of $5 \cdot 10^{-5}$ sec. Hence, a measurement can be made immediately after connecting or disconnecting the source of heat, the time taken to carry out the experiments thereby being reduced to several seconds. The method enables one to study the development of convective heat transfer at high heating or cooling rates, since the resolving power of the frequency method increases when the rate of variation of the temperature is increased (the nonstationary mode), while the relative error in measuring the frequency shift is reduced. Among the features of the method is the fact that information on the

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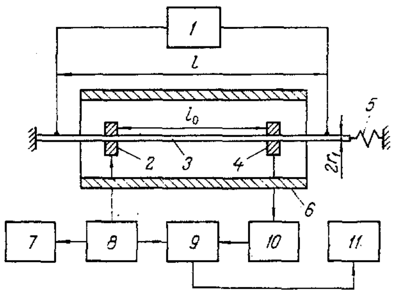

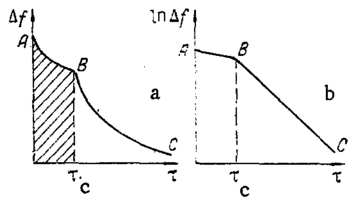

Fig. 1 Fig. 2

Fig. 1. Block diagram of the apparatus; 1) source of heat; 2, 4) source and receiver of acoustic oscillations; 3) wire probe; 5) spring; 6) confining tube; 7) frequency meter; 8) master generator; 9) frequency deviation measurer; 10) amplifier; 11) oscilloscope.

Fig. 2. Curves of Δf as a function of τ (a) and $\ln\Delta f$ as a function of τ (b), which occurs when the wire probe is heated: The section AB corresponds to conductive heat transfer, and BC corresponds to convective heat transfer; τ_c is the delay time.

variation of the temperature is supplied from the probe in the form of a frequency-modulated signal, and not an amplitude-modulated signal as in [1-9, 14, 15]. In the latter case, to increase the sensitivity, it is necessary to reduce the passband of the amplifier, which imposes limitations on the range of variation of the measured parameter, i.e., on the investigation of the dynamics of the process.

Consider the temperature field produced by a thin wire through which an electric current is passed (Fig. 1). Simultaneously, continuous acoustic oscillations of constant frequency f_0 are excited at one end of the wire. When the wire is heated the frequency of the oscillations received at the opposite end will differ from the frequency of the excited signals, i.e., a nonstationary acoustic Doppler effect occurs, the value of which is proportional to the rate of heating of the wire [12]

$$\Delta f(\tau) = f - f_0 = A\frac{dT}{d\tau}. \quad (1)$$

If the wire is situated in a confined space, for example, stretched along the axis of a cylindrical tube, when the wire is heated the heat will be preferentially transferred initially into the surrounding liquid or gas due to thermal conduction. Then, after a certain time τ_r, which Ostroumov [6] has called the delay time, free convection begins to have a considerable effect on the heat-transfer process. Since for a given initial rate of heating of the wire and an assigned value of the diameter of the confining cylinder there is a definite value of the fall in temperature of the wire ΔT_c corresponding to the instant of time τ_c, the parameters T_c and ΔT_c can be called critical or threshold values characterizing the free convection. Since the intensity of convective heat transfer in media of low viscosity considerably exceeds the intensity of conductive heat transfer, to simplify the calculations of the heating of the wire probe we can neglect thermal conduction under free convection conditions.

We will calculate the heating of the wire for the first stage when there is heat conduction, and for the second stage, when we will assume only convective heat transfer. The thermal balance equation for the wire in the first stage will be

$$\pi r_1^2 l c\gamma\frac{dT}{d\tau} = P + 2\pi r_1 l\lambda_g\frac{dT_g}{dr}. \quad (2)$$

To simplify the problem we will neglect the volume thermal capacity of the medium surrounding the wire, which is justified in the case of the gas. Then, for steady-state radial thermal flow through the gaseous medium in the region $r_1 \le r \le r_2$, the differential equation of thermal conduction has the form

$$\frac{d}{dr}\left(r\frac{dT_g}{dr}\right) = 0. \quad (3)$$

If the temperature of the surrounding cylinder is maintained constant and equal to T_0 and the temperature of the wire is T, the initial and boundary conditions will be

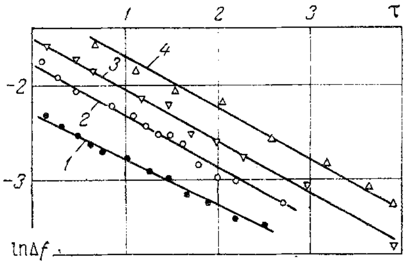

Fig. 3. Experimental curves of $\ln\Delta f$ versus τ (sec) when the wire is heated in open air: 1) P = 0.2 W; 2) 0.3 W; 3) 0.4; 4) 0.5 W.

$$T(r,\ 0) = T_0;\ T_g(r_1,\ \tau) = T;\ T_g(r_2,\ \tau) = T_0. \tag{4}$$

The solution of Eqs. (2) and (3) with condition (4) for an excess temperature $\Delta T(\tau) = T(\tau) - T_0$ can be written

$$\Delta T(\tau) = \frac{\ln r_2/r_1}{2\pi l} \frac{P}{\lambda_g} [1 - \exp(-\lambda_g h\tau)], \tag{5}$$

where

$$h = \frac{2}{c\gamma r_1^2 \ln r_2/r_1} \tag{6}$$

is a constant coefficient that depends on the properties of the wire and the internal radius of the surrounding tube.

Finding from Eq. (5) the rate of heating of the wire probe and substituting the value obtained into the expression for the frequency shift (1), we obtain

$$\Delta f(\tau) = A_1 \exp(-\lambda_g h\tau), \tag{7}$$

where

$$A_1 = A \frac{P}{\pi r_1^2 l c\gamma} \tag{8}$$

is a constant which is also determined by the properties of the probe and the electric power supplied to it. Taking logarithms of Eq. (7), we have

$$\ln \frac{\Delta f(\tau)}{A_1} = -\lambda_g h\tau. \tag{9}$$

Hence, on a semilogarithmic scale the experimental curve $\Delta f(\tau)$ is a straight line (Fig. 2b), from the slope of which

$$k_1 = -\lambda_g h, \tag{10}$$

one can directly calculate the thermal conductivity of the gas (liquid) surrounding the probe:

$$\lambda_g = -\frac{k_1}{h} = -\frac{1}{2} k_1 c\gamma r_1^2 \ln r_2/r_1. \tag{11}$$

When the wire is heated conditions are produced for the occurrence and development of free convection, to which corresponds the instant of time τ_c, so that taking Eq. (5) into account we can write

$$\Delta T_c = \frac{\ln r_2/r_1}{2\pi l} \frac{P}{\lambda_g} [1 - \exp(-\lambda_g h\tau_c)]. \tag{12}$$

From Eq. (1) the threshold temperature drop can be calculated from the equation

$$\Delta T_c = \frac{1}{A} \int_0^{\tau_c} \Delta f(\tau)\, d\tau. \tag{13}$$

For the second stage of the heating of the wire probe (convective heat transfer) the heat-balance equation will be (for $\tau_c \le \tau \le \infty$)

$$\pi r_1^2 l c\gamma \frac{d(\Delta T)}{d\tau} = P - 2\pi r_1 l\alpha\Delta T. \tag{14}$$

As in the previous case, we will assume that the initial temperature of the medium surrounding the wire is constant and equal to the temperature of the external cylinder T_0. The initial condition is

$$\Delta T(r, \tau_c) = \Delta T_c. \tag{15}$$

The boundary conditions are the same as in Eq. (4). The solution of problem (14) taking Eqs. (4) and (15) into account has the form

$$\Delta T(\tau) = \frac{P}{S\alpha} + \left(\Delta T_c - \frac{P}{S\alpha}\right)\exp\left[-\alpha m(\tau - \tau_c)\right], \tag{16}$$

where

$$m = \frac{2}{c\gamma r_1} \tag{17}$$

is a constant coefficient determined by the parameters of the probe.

Performing with expression (16) the same transformation as with (5), we obtain for the period of convective heat transfer

$$\Delta f(\tau) = A_2 \exp(-\alpha m\tau). \tag{18}$$

Here

$$A_2 = A\left(\frac{Pm}{S} - \alpha m\Delta T_c\right)\exp(\alpha m\tau_c). \tag{19}$$

Equation (18) is the main equation for the experimental determination of the heat transfer coefficient from the measured time-variation of the frequency shift of acoustic oscillations. To do this we take the logarithm of this expression:

$$\ln\frac{\Delta f(\tau)}{A_2} = -\alpha m\tau. \tag{20}$$

Relation (20) for convective heat transfer is also described by a straight line (Fig. 2b), but having a different slope given by

$$k_2 = -\alpha m. \tag{21}$$

By finding the quantity k_2 we can find the heat-transfer coefficient from the equation

$$\alpha = -\frac{k_2}{m} = -\frac{1}{2}k_2 c\gamma r_1. \tag{22}$$

The delay time τ_c corresponding to the occurrence of free convection is determined from the discontinuity in the experimental curves of $\Delta f(\tau)$ or $\ln\Delta f(\tau)$ (Fig. 2). The value of ΔT_c is numerically equal to the area shown hatched in Fig. 2a.

It should be noted that the above method is nonstationary. By varying the power applied to the probe one can obtain different rates of heating of the wire filament and hence investigate how the threshold for the development of free convection and the heat transfer coefficient depend on the rate of heating.

As is seen from Eqs. (11) and (22), the accuracy with which the thermal conductivity and the heat-transfer coefficient can be determined depends only on the accuracy of the graphical determination of the angular coefficients k_1 and k_2, and the accuracy with which the parameters of the wire and of the surrounding tube (c, γ, r_1 and r_2) are assigned, and ideally are independent of other quantities including the acoustic characteristics (the velocity of ultrasound in the probe), its temperature coefficient, etc.

To realize this method in practice we constructed the apparatus shown in Fig. 1, for recording the rate of heating of a wire probe. The frequency shift between the oscillations at the receiver and radiator when the wire is heated was measured with a Ch7-5 frequency comparator and was recorded on an N700 oscilloscope. The frequency stability of the master generator of electrical oscillations was monitored with a Ch3-30 electronic-counter frequency meter.

TABLE 1. Experimentally Obtained Values of α, λ_g, and τ_c for Different Heat-Transfer Conditions

Tube diameter $2r_2$, mm	Initial temperature T_0, °K	Power, P, W	α, W/m²·deg	$\lambda_g \cdot 10^3$, W/m·deg	τ_c, sec
Open air	294	0,2 0,3 0,4 0,5	87 99 100 102		
8	291	0,1 0,3	116	25	1,1—1,6
14	295	0,1 0,3	115	26	0,8—1,1

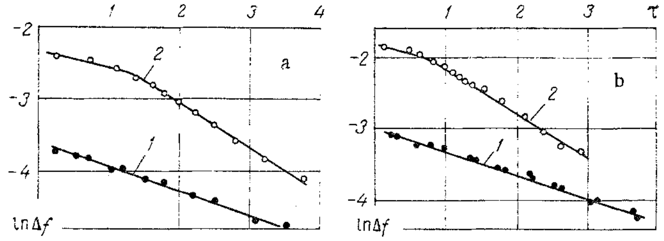

Fig. 4. Experimental curve of $\ln\Delta f$ as a function of τ (sec) for the heating of a filament in a tube with internal diameter 8 mm (a) and 14 mm (b): 1) P = 0.1 W; 2) P = 3 W.

We used a Constantan wire of diameter 0.2 mm and length 412 mm as the probe. This has the minimum temperature coefficient of resistance and simplifies the supply of constant power to the probe which is one of the advantages of the method. The filament was heated by constant current and placed along the axis of a horizontal metal tube.

The use of the Ch7-5 frequency comparator, constructed on the principle of carrier frequency multiplication, enabled us to measure small frequency shifts due to a change in the temperature of the filament with a low absolute error of the order of 10^{-2} Hz. The sensitivity of the acoustic frequency method for measuring the rate of heating (cooling) is then 0.005 deg/sec.

Experimental investigations of the nonstationary heat transfer of a fine wire under free convection conditions were made in open air and in surrounding cylinders of diameters 8mm and 14 mm for different filament heating powers. All the measurements were made under identical laboratory conditions: at initial room temperature and under normal atmospheric pressure.

The experimental investigations showed that when the wire is heated in open air (Fig. 3) convective heat transfer begins almost immediately after the source of heat is switched on. The straight lines are almost parallel to one another; their slope increases slightly as the heating power is increased, which indicates some increase in the heat-transfer coefficient as the rate of heating increases (see the table).

The nature of the heat transfer changes when the probe is placed in a confining horizontal tube. In all cases, for a low heating power of the order of 0.1 W the experimental curve does not have a discontinuity (straight lines 1 in Fig. 4). In this case convection has practically no effect and the heat transfer in air occurs primarily due to the thermal conductivity of the gas. A calculation of the thermal conductivities in these experiments gave results in good agreement with the available data [2, 13]: $\lambda_g = 25.4 \cdot 10^{-3}$ W/m·deg for T = 290°K, and $\lambda_g = 26.2 \cdot 10^{-3}$ W/m·deg for T = 300°K.

For a power P = 0.3 W a break is observed in the curve of $\ln\Delta f(\tau)$ which represents the onset of convective heat transfer (curves 2 in Fig. 4). As might have been expected, free convection begins earlier when the diameter of the surrounding tubes increased. It can be assumed that before the break occurs in the experimental curve conductive heat transfer occurs, and after this point convective heat transfer. The value of the Rayleigh criterion for these cases varies within the limits Ra = 100-300.

The values of Ra obtained are explained by the nonstationary nature of the process and the geometry of the system, since theoretically there is no unstable equilibrium in the cylindrical system (if the filament is absolutely thin) and there should not be a convection threshold.

The results of experimental investigations for λ_g, α, and τ_c are given in Table 1.

The relative theoretical error in determining the heat-transfer coefficient was 4.3%, and the error in determining the thermal conductivity was 5.5%. The accuracy achieved is not the limiting value and can be considerably improved by increasing the multiplication factor of the carrier frequency of the signals and by more modern methods of processing the experimental oscillograms.

The solutions and theoretical expressions given in this paper were obtained assuming that the heat-transfer coefficients are constant. Constancy of the quantity α leads to the linear relationship $\ln\Delta f\,(\tau)$, so that the equations given above can only be used on the linear part of this curve. The solution of the problem of convective heat transfer with variable coefficient α has not been given since the nature of the variation of the coefficient as a function of the rate of heating the thermal flux, and the time is not known.

In addition, in a more rigorous formulation of the problem it is necessary to take into account the temperature field inside the wire, which does not give rise to any great difficulties, but leads to a complication of the theoretical formulas. In the case of liquids and gases at high pressures it is necessary to take into account the heat capacity of these media. The error due to neglecting these factors is negligibly small in our experiments.

Further investigations in this direction will enable us to study the mechanism of heat transfer in liquids and gases in more detail.

NOTATION

τ, time; r, radius of the cylindrical system of coordinates; f_0, f, frequencies of the source and receiver of acoustic oscillations, respectively; t, T_g, temperatures of the wire and surrounding medium (gas), respectively; A, acoustic constant quantity; r_1, radius of the wire; r_2, internal radius of the surrounding tube; l, length of the heated part of the wire; c, γ, specific heat and density of the wire; P, electric power used to heat the wire; T_0, initial temperature of the wire; λ_g, thermal conductivity of the gas (liquid); α, heat-transfer coefficient; $S = 2\pi r_1 l$, area of the surface of the heated wire.

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